Pricing and bandwidth allocation problems in wireless multi-tier networks

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***Abstract*—Future cellular networks are facing crucial architecture changes to cope with high throughput, energy and cost-efficiency demands. Emerging solutions are small-cells and femto-cells which will coexist with classical macro-cells technology. In these heterogeneous networks, we study the joint service pricing and bandwidth allocation problem at the operator level. Each user selfishly adopts the service that optimizes its satisfaction. The user-level problem is formulated as a general non-atomic game. The Wardrop equilibrium is proven to exist and an analytical expression is provided for arbitrary number of services. The equilibria multiplicity, the influence of pricing and bandwidth allocation policies are investigated numerically.**

***Index Terms*—Small-cells, femto-cells, hierarchical games, Wardrop equilibrium, Stackelberg equilibrium**

I. Introduction

The constantly increasing demand for higher data rate services motivates the development of new communication standards and network architectures. Among the candidate solutions for the next generation wireless networks are small-cell [1] and femto-cells [2], [3] dedicated to outdoor and indoor services respectively. Small-cells are short-range cells densely deployed which enhance the throughput by increasing the spectral reuse and decreasing the transmitter-receiver distance. Femto-cells are used for traffic offloading and indoor coverage and can be connected to the network via DSL, cable modems or orthogonal radio bandwidths. In such heterogeneous networks, several issues arise: what is the energy and cost-efficient way in which the operator should deploy, inter-connect, allocate resources among these networks (i.e., spectrum allocation, interference management), and price the different types of provided services? Our objective, in this paper, is to address some of these issues using game-theoretical tools.

The most relevant related works are [4], [5], [6]. All these works use the framework of hierarchical non-cooperative games. In [4], the users' dynamic subscription problem is studied for a network where two competing operators chose their own service price. At a lower level, the users can choose among three options: adopt a service with a constant QoS, a service with a variable QoS (depending on the fraction of users that adopt the same service), or do not adopt any service. The Nash equilibrium of the duopoly competition and the equilibrium in the non-atomic congestion game are derived and analysed. In [6], the system is composed of an operator providing two different services. The operator objective is to choose the service prices in order to maximize its revenue. The revenue at equilibrium point and the optimal prices are analysed via numerical simulations. In [5], a three-level hierarchical game is studied. At the higher level, the operators decide which technology to adopt (among 3G, WiFi and WiMax); at the intermediate level, the operators maximize their revenues by choosing the prices for the provided services; at the lower level, a congestion non-atomic game is analysed where the users choose the service depending on their Quality of Service and price.

The present paper can be seen as an extension of [6]. However, our contributions are significant and multi-fold : (i) we rigorously define and study the two-level Stackelberg formulation; (ii) we consider an arbitrary number of services, prove the existence of the Wardrop equilibrium (WE) and give its analytic expressions in the low-level non-atomic game; (iii) we illustrate the existence of multiple WE via numerical simulations; (iv) we consider the bandwidth allocation problem among the two technologies: macro-cells and small-cells; (v) throughput models and numerical simulations are provided for three types of services: *macro-cells* only (M), *macro-plus-small-cells* (MS) and *macro-plus-small-plus-femtocells* (MSF); (vi) we analyse the influence of prices and bandwidth allocation policy on the network state; (vii) we provide analytical expressions for the maximum service prices (s.t. beyond these bounds, the service will not be adopted by any user). The most general case, in which an arbitrary number of operators provide each an arbitrary number of services is left as an interesting extension of our work and [4].

This work is organized as follows. Our system model is described in Sec. II. In Sec. III, we state our main result: the existence of SE in the bi-level game. In Sec. IV, we focus on the particular case where only three services are provided. First, we model the throughputs, and, then, we illustrate, via numerical simulations, the influence of pricing and bandwidth allocation policy. We conclude in Sec. V.

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II. System Model

We consider a heterogeneous wireless network where the operator, i.e., the network owner, deploys a number of $T \geq 1$ technologies, such as: Macro-Cells (MC), Small-Cells (SC) and Femto-Cells (FC). The operator is assumed to provide to its users a number of $S \geq 2$ different services based on these technologies. We assume an infinite number of users $N \to +\infty$. These users are divided in an infinite number of different classes $[0, \gamma_{\max}]$ depending on their valuation or satisfaction w.r.t. their experienced quality of service (QoS). We define by $\Gamma : [0, \gamma_{\max}] \to [0, 1]$ the distribution function of the population of users in function of these classes.

Notations: We denote by $\mathcal{S} = \{1, \ldots, S\}$ the set of available services. The vector $(\boldsymbol{\alpha}, x) \in [0, 1]^{S+1}$ stands for the network state, i.e., the vector of loads in the network such that:

- $x \in [0, 1]$ is the fraction of the population that adopts some service;
- $\alpha_s x \in [0, 1]$ is the fraction of the population that adopts service $s \in \mathcal{S}$; $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_S)$ and $\sum_{s=1}^{S} \alpha_s = 1$.

At the operator level, the degrees of freedom are $(\mathbf{p}, \boldsymbol{\beta})$, where $\mathbf{p} = (p_1, \ldots, p_S) \in [0, P]^S$ is the vector of prices charged for the provided services, and $\boldsymbol{\beta} \in \mathcal{B}$ with $\mathcal{B} = \left\{(\beta_1, \ldots, \beta_T) \in [0, 1]^T | \sum_{k=1}^{T} \beta_k = 1\right\}$ is the bandwidth allocation policy vector among the deployed technologies assumed to be operating in orthogonal frequency bands.

III. Joint pricing and bandwidth allocation problem in a multi-tier heterogeneous network

In this section, we analyse the joint pricing and bandwidth allocation problem in a heterogeneous network as a bi-level Stackelberg game [7]. The *leader* of the game, i.e., the system operator, chooses the vector of service prices and the bandwidth allocation policies, $(\mathbf{p}, \boldsymbol{\beta})$, to optimize its performance criteria. Then, given the choice of the leader, the *followers*, i.e., the customers, play a non-atomic non-cooperative game, by selfishly choosing the service that optimizes their individual satisfaction. The mutual interference that is created in the system gives rise to an interactive situation among the selfish users.

A. General non-atomic game

The *low-level* problem, the non-atomic game is defined by:

- The *infinite population of players* divided in classes $[0, \gamma_{\max}]$;
- The *set of actions* is the set $\mathcal{S}^* = \{0\} \cup \mathcal{S}$;
- The *payoff functions* $\{U_s^\gamma(\cdot)\}_{s \in \mathcal{S}^*, \gamma \in [0, \gamma_{\max}]}$, where, similarly to [6], the payoff function of a user of class γ obtained by adopting service s is given by:

$$U_s^\gamma(\boldsymbol{\alpha}, x, \boldsymbol{\beta}, \mathbf{p}) = \gamma g_s(\boldsymbol{\alpha}, x, \boldsymbol{\beta}) - p_s, \qquad (1)$$

  where $g_s(\boldsymbol{\alpha}, x, \boldsymbol{\beta})$ represents the experienced QoS or throughput when adopting service s and which depends on the network state $(\boldsymbol{\alpha}, x)$ and on the bandwidth allocation vector $\boldsymbol{\beta}$ with $g_0 \equiv 0$ and $p_0 = 0$ (the benefit and price for not adopting any service is zero).

The solution concept of a non-atomic game is the WE. Intuitively, the WE is the equivalent of the Nash equilibrium in a population game. It is a state of the network which is robust to the deviation of an infinitesimal fraction of a population. This concept has been introduced by Wardrop in the context of transportation routing problems [8] and defined by the two principles:

- *The journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route.*
- *At equilibrium the average journey time is minimum.*

We observe the analogy between the provided services, users, payoff functions and routes, vehicles, journey times, respectively.

Remark 3.1: The payoff obtained when connected to service s depends on all the service loads in the system and not only on the load in service s. Therefore, our framework is more general than that of crowding games [9]. As we will see in the next section, this consideration is crucial in the study of co-existent technologies such as MC, SC and FC.

Our main result is given in the following theorem.

Theorem 3.1: In the non-atomic game described above, $\mathcal{G} = \left\{[0, \gamma_{\max}], \mathcal{S}^*, \{U_s^\gamma\}_{s \in \mathcal{S}^*, \gamma \in [0, \gamma_{max}]}\right\}$, for fixed $\mathbf{p} \in [0, P]^S$ and $\boldsymbol{\beta} \in \mathcal{B}$, if the following two conditions are met:

[C1] *the payoff functions $\{U_s^\gamma\}_{s,\gamma}$ depend only on the loads, i.e., $(\boldsymbol{\alpha}, x) \in [0, 1]^{S+1}$ and not on the users' identities;*

[C2] *the functions $g_s(\boldsymbol{\alpha}, x, \boldsymbol{\beta})$ are continuous w.r.t. $(\boldsymbol{\alpha}, x)$.*

Then there exists at least one Wardrop equilibrium.

The proof follows from [**C1**], [**C2**] and applying Theorem 2 in [10] to our scenario.

Theorem 3.2: In the non-atomic game described in the previous theorem, if the following condition is also satisfied:

[C3] *there exists an ordering such as, $\forall(\boldsymbol{\alpha}, x)$:*
$g_{r_1}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}) < g_{r_2}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}) < \cdots < g_{r_S}(\boldsymbol{\alpha}, x, \boldsymbol{\beta})$, .

Then the equilibrium points are solutions of the following fixed-point system of equations:

$$\begin{cases} \alpha_s^{WE} & = & 0, \ \forall s \ \in \mathcal{S} \setminus \{c_1, \ldots, c_Q\} \\ \hat{\gamma}_{c_j} & = & \frac{p_{c_j} - p_{c_{j-1}}}{g_{c_j}(\boldsymbol{\alpha}^{WE}, x^{WE}, \boldsymbol{\beta}) - g_{c_{j-1}}(\boldsymbol{\alpha}^{WE}, x^{WE}, \boldsymbol{\beta})}, \\ & & \forall \ j \in \{1, \ldots, Q\} \\ x^{WE} & = & 1 - \Gamma(\hat{\gamma}_{c_1}) \\ \alpha_{c_j}^{WE} & = & \frac{\Gamma(\hat{\gamma}_{c_{j+1}}) - \Gamma(\hat{\gamma}_{c_j})}{x^{WE}}, \ \forall \ j \in \{1, \ldots, Q\} \end{cases} \qquad (2)$$

where $0 \leq \hat{\gamma}_{c_1} \leq \hat{\gamma}_{c_2} \leq \cdots \leq \hat{\gamma}_{c_Q} \leq \gamma_{max}$ are thresholds on the classes of users such that, at the WE all the users of type $\gamma \in (\hat{\gamma}_{c_k}, \hat{\gamma}_{c_{k+1}}]$ connect to service c_k for all $k \in \{0, 1, \ldots, Q\}$, with the convention that $\hat{\gamma}_{c_0} = 0$ and $\hat{\gamma}_{c_{Q+1}} = \gamma_{\max}$. Depending on the prices $\mathbf{p}$, only a subset of $Q \leq S$ services, i.e. $\{c_1, \ldots, c_Q\}$, will be adopted at the WE. This subset is such that the two conditions are met simultaneously:

$$\begin{array}{lllll} g_{c_1}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}) & < & \cdots & < & g_{c_Q}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}) \quad \forall(\boldsymbol{\alpha}, x) \\ p_{c_1} & \leq & \cdots & \leq & p_{c_Q}. \end{array} \qquad (3)$$

Condition [**C3**] simply means that the operator guarantees an experienced QoS for each service which is independent on the system loads. For an intuitive insight, consider two services s_i and s_j such that $p_{s_i} \geq p_{s_j}$ and $g_{s_i}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}) < g_{s_j}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}), \quad \forall\ (\boldsymbol{\alpha}, x)$, i.e., service s_i is priced higher in spite of the fact that it provides a lower throughput than s_j. It is obvious that service s_i will not be chosen by any rational user (or payoff maximizer).

B. Optimal pricing and bandwidth allocation policies

At the *higher level*, the operator chooses the service prices and optimal bandwidth allocation policy to optimize the mean revenue per user given by:

$$\max_{\mathbf{p} \in [0,P]^S, \boldsymbol{\beta} \in \mathcal{B}} \left\{ \sum_{s=1}^{S} p_s\ \alpha_s^{WE}(\mathbf{p}, \boldsymbol{\beta}) x^{WE}(\mathbf{p}, \boldsymbol{\beta}) \right\} \tag{4}$$

Assuming that the operator is aware of the network state at the WE, then it is able to choose the prices that maximize its revenue. From theorem 3.1 we have the existence of at least one WE for any pricing policy $\mathbf{p} \in [0, P]^M$ and any bandwidth allocation policy in $\boldsymbol{\beta} \in \mathcal{B}$. We also have that the feasible set (i.e., $[0, P]^S \prod \mathcal{B}$) is a compact and convex set. However, since multiple WE might exist for certain values of $(\mathbf{p}, \boldsymbol{\beta}) \in [0, P]^S \prod \mathcal{B}$, we consider that the operator chooses either the worst or the best equilibrium. These well-defined optimization problems will be studied via numerical simulations where the feasible set will be a finite quantization of $[0, P]^S \prod \mathcal{B}$ ensuring that an optimal point $(\mathbf{p}^*, \boldsymbol{\beta}^*)$ exists.

IV. Specific Case: Three Available Services

In this section, we consider a particular scenario where the operator deploys three technologies MC, SC and FC. Based on these technologies, the operator provides to its customers three different services: macro-cell service M, macro-plus-small-cells service MS and macro-plus-small-plus-femto-cells service MSF.

A. Model of the throughput functions

The model for the throughput functions $g_s(\cdot)$ is similar to the one proposed in [6] for the MC and FC system. Here, we extend this model by including the SC technology and changing accordingly the services provided by the operator. We assume that inside each MC we have both SC and FC but in different areas.

The throughputs are defined as:

$$\begin{aligned} g_{\text{M}}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}) &= \mathbb{E}[T_{mc}] \\ g_{\text{MS}}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}) &= \tau_{sc}\mathbb{E}[T_{sc}] + (1-\tau_{sc})\mathbb{E}[T_{mc}] \\ g_{\text{MSF}}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}) &= \tau_{fc}\mathbb{E}[T_{fc}] + \tau_{sc}\mathbb{E}[T_{sc}] + \tau_{mc}\mathbb{E}[T_{mc}] \end{aligned} \tag{5}$$

where τ_{fc}, τ_{sc}, τ_{mc} are the expected fractions of time that users spend indoors, in areas with small-cell coverage, and in other areas respectively; $\mathbb{E}[T_{fc}]$, $\mathbb{E}[T_{sc}]$, $\mathbb{E}[T_{mc}]$ are the downlink throughputs obtained when connected to a FC, SC or MC respectively.

The following two assumptions are made:

Hypothesis 4.1: $\tau_{mc} < \tau_{sc} < \tau_{fc}$

Hypothesis 4.2: $\mathbb{E}[T_{mc}] < \mathbb{E}[T_{sc}] < \mathbb{E}[T_{fc}]$ ensuring that the throughputs $g_s(\cdot)$ are ordered as:

$$g_{\text{M}}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}) < g_{\text{MS}}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}) < g_{\text{MSF}}(\boldsymbol{\alpha}, x, \boldsymbol{\beta}). \tag{6}$$

The intuition behind *Hypothesis 4.2* is the that MC sustain all the users adopting services that employ also SC and FC in the areas covered only by MC. Similarly, the SC sustain the users that employ also FC in the areas covered by SC.

We further assume that, inside each MC, the users are identically distributed over the available services as in the whole network. Also, at a MC base station level, the arrivals of individual user requests and the service times of these requests are Poisson random processes. Then, the total packet rate request at a MC base station is given by:

$$\lambda_{mc} = \lambda x \left[\alpha_{\text{M}} + (1-\tau_{sc})\alpha_{\text{MS}} + (1-\tau_{sc}-\tau_{fc})\alpha_{\text{MSF}}\right] N_{\text{cell}},$$

where $\lambda x \alpha_{\text{M}} N_{\text{cell}}$ is the rate of packet request from all users inside the MC adopting service M; $(1-\tau_{sc})\lambda x \alpha_{\text{MS}} N_{\text{cell}}$ is the rate from users adopting service MS that are not SC coverage area; $(1-\tau_{sc}-\tau_{fc})\lambda x \alpha_{\text{MSF}} N_{\text{cell}}$ is the rate from users adopting service MSF that are neither indoors nor in SC coverage areas.

The rate at which the MC base station (BS) serves the incoming packet requests is $\mu_{mc} = \frac{\beta_{mc} R_{mc}}{L}$ where $\beta_{mc} R_{mc}$ is the achievable rate of the downlink channel (rate at which the BS can send reliable information to all the users) and L is the mean file length.

We assume that $\lambda_{mc} < \mu_{mc}$ and, denoting $\rho_{mc} = \lambda_{mc}/\mu_{mc}$, we have

Hypothesis 4.3: $0 < \rho_{mc} < 1$.

Following a similar proof to the one in [6] and assuming

Hypothesis 4.4: $\lim_{N_{\text{cell}} \to \infty} \dfrac{\mu_{mc}}{\lambda N_{cell}} = c_{mc}$

where $0 < c_{mc} < \infty$, we obtain:

$$\mathbb{E}[T_{mc}] = \beta_{mc} R_{mc} \left[1 - \rho_{mc}\right] \frac{-\log(1-\rho_{mc})}{\rho_{mc}}. \tag{7}$$

with $\rho_{mc} = \dfrac{[\alpha_{\text{M}} + (1-\tau_{sc})\alpha_{\text{MS}} + (1-\tau_{sc}-\tau_{fc})\alpha_{\text{MSF}}]x}{c_{mc}}$.

We proceed in a similar way to obtain the expected small-cell throughput $\mathbb{E}[T_{sc}]$. Here, the total service rate request at a SC base station is:

$$\lambda_{sc} = \tau_{sc} \lambda x (\alpha_{\text{MS}} + \alpha_{\text{MSF}}) \frac{N_{\text{cell}}}{A_{\text{cell}}} \tag{8}$$

where $\tau_{sc} \lambda x \alpha_{\text{MS}} N_{\text{cell}}/A_{\text{cell}}$ is the rate of packet request from users adopting service MS that are inside the SC; $\tau_{sc} \lambda x \alpha_{\text{MSF}} N_{\text{cell}}/A_{\text{cell}}$ is the rate request of users choosing MSF service that are inside the SC. The term A_{cell} denotes the number of SC inside a MC.

The rate at which the SC base station (BS) serves the incoming packet requests is $\mu_{sc} = \frac{\beta_{sc} R_{sc}}{L}$ where $\beta_{sc} R_{sc}$ denotes the rate at which the small-cell can send reliable information to all the users. We assume that $\lambda_{sc} < \mu_{sc}$, and denoting $\rho_{sc} = \frac{\lambda_{sc}}{\mu_{sc}}$, we have:

Hypothesis 4.5: $0 < \rho_{sc} < 1$.

By considering:

Hypothesis 4.6: $\lim_{N_{\text{cell}} \to \infty} \dfrac{\mu_{sc} A_{\text{cell}}}{\lambda N_{\text{cell}}} = c_{sc}$

where $0 < c_{sc} < \infty$ is a positive constant, we obtain:

$$\mathbb{E}[T_{sc}] = \beta_{sc} R_{sc} \left[1-\rho_{sc}\right] \frac{-\log(1-\rho_{sc})}{\rho_{sc}}. \quad (9)$$

with $\rho_{sc} = \dfrac{[1-\alpha_{\text{M}}]\tau_{sc}x}{c_{sc}}$.

For simplicity sake, we model the throughput obtained when connecting to an indoor FC as being proportional to R_{mc}, i.e., $E[T_{fc}] = \kappa_{fc} R_{mc}$.

To conclude, we normalize the throughput functions, the prices and the operator revenue by R_{mc} and obtain:

$$\frac{g_{\text{M}}(\boldsymbol{\alpha}, x, \boldsymbol{\beta})}{R_{mc}} = \beta_{mc}\varphi_{mc}$$
$$\frac{g_{\text{MS}}(\boldsymbol{\alpha}, x, \boldsymbol{\beta})}{R_{mc}} = \tau_{sc}\beta_{sc}\frac{R_{sc}}{R_{mc}}\varphi_{sc} + (1-\tau_{sc})\beta_{mc}\varphi_{mc}$$
$$\frac{g_{\text{MSF}}(\boldsymbol{\alpha}, x, \boldsymbol{\beta})}{R_{mc}} = \tau_{fc}\kappa_{fc} + \tau_{sc}\beta_{sc}\frac{R_{sc}}{R_{mc}}\varphi_{sc} + \tau_{mc}\beta_{mc}\varphi_{mc}$$

where $\varphi_{mc} = [1-\rho_{mc}]\frac{-\log(1-\rho_{mc})}{\rho_{mc}}$, $\varphi_{sc} = [1-\rho_{sc}]\frac{-\log(1-\rho_{sc})}{\rho_{sc}}$.

In order to ensure that *Hypothesis 4.2* is met, we have the following constraints on the system parameters: $c_{sc} > \tau_{sc}/\tau_{mc}$, $c_{mc} > 1$, $\beta_{sc}R_{sc}/R_{mc} < \kappa_{fc}$.

Remark 4.1: All the aforementioned conditions and hypothesis are required to guarantee a minimum experienced QoS dictated by (6). This means that, in order to provide enhanced services (e.g., MS, MSF), some resources must be allocated to the new technologies SC, FC.

Given the above model of throughputs, all the conditions in Theorems 3.1 and 3.2 are met therefore the existence of a WE is ensured. The WE is given by (2).

B. Price upper-bounds

Depending on the system parameters, there exist certain maximum values for the service prices such that, above these values, no user will adopt these services. For service M, the maximum price is given by the solution to the equation $\max\limits_{\gamma,\varphi_{mc},\boldsymbol{\alpha},x}[\gamma g_{\text{M}}(\boldsymbol{\alpha}, x, \boldsymbol{\beta})] - p_{\text{M}} = 0$ and is equal to $p_{\text{M}}^{max} = \gamma_{max}\beta_{mc}$. The maximum prices for services MS and MSF follow similarly: $p_{\text{MS}}^{max} = \gamma_{max}\left[\tau_{sc}\beta_{sc}R_{sc}/R_{mc} + (1-\tau_{sc})\beta_{mc}\right]$, $p_{\text{MSF}}^{max} = \gamma_{max}\left[\tau_{fc}\kappa_{fc} + \tau_{sc}\beta_{sc}\dfrac{R_{sc}}{R_{mc}} + \tau_{mc}\beta_{mc}\right]$.

C. Numerical simulations

First, we consider the case of fixed bandwidth allocation, i.e., $\beta_{mc} = 0.43$, $\beta_{sc} = 0.4$, $\beta_{fc} = 0.17$. The following scenario was investigated: $\gamma_{max} = 1$, $\tau_{mc} = 0.15$, $\tau_{sc} = 0.25$, $\tau_{fc} = 0.6$, $R_{sc}/R_{mc} = 1.79$, $\kappa_{fc} = 2$. The price of service MSF is fixed $p_{\text{MSF}} = 0.7$. Using the upper-bounds in Subsec. IV-B, the maximum prices are $p_{\text{M}}^{max} = 0.43$, $p_{\text{MS}}^{max} = 0.5$ and $p_{\text{MSF}}^{max} = 1.44$.

Fig. 1 illustrates the operator revenue as a function of $p_{\text{M}} \in \{0, 0.1, \cdots, 2\}$ and $p_{\text{MS}} \in \{0, 0.1, \cdots, 2\}$. In order to obtain the operator revenue, all the WE points were numerically computed by solving the system (2). It turns out that there may exist several WE. In our plots, unless otherwise specified, we always choose the point that yields the minimum operator revenue (worst-case WE). The discontinuities in Fig. 1 are explained by the multiplicity of the WE and by the values of the upper-bounds $p_{\text{M}}^{max} = 0.43$, $p_{\text{MS}}^{\max} = 0.5$. The fractions of users that adopt service M, MS, MSF ($\alpha_{\text{M}}^{WE}x^{WE}$, $\alpha_{\text{MS}}^{WE}x^{WE}$, and $\alpha_{\text{MSF}}^{WE}x^{WE}$) are plotted in Fig. 2, 3, 4 respectively. In these figures, we observe that if $p_M \geq p_M^{max}$ and $p_{MS} \geq p_{MS}^{max}$ then at the WE only service MSF will be used and the WE is unique. Otherwise, there may be multiple WE and this explains the irregularities in Fig. 2, 3. By tuning the prices, the operator can manipulate the system loads at the WE.

Second, we no longer assume a fixed bandwidth allocation policy but $\beta_{mc} \in \{0.03, 0.08, 0.13 \cdots, 0.73\}$, $\beta_{fc} = 0.17$, $\beta_{sc} = 1 - \beta_{mc} - \beta_{fc}$.[1] The following scenario was considered: $\gamma_{max} = 1$, $\tau_{mc} = 0.15$, $\tau_{sc} = 0.25$, $\tau_{fc} = 0.6$, $R_{sc}/R_{mc} = 12.17$, $\kappa_{fc} = 11$.

In Fig. 5, we plot the operator revenues at the worst and best WE as function of β_{mc} for the prices $p_{\text{M}} = 0.5$, $p_{\text{MS}} = 2.0$ and $p_{\text{MSF}} = 4.2$. We observed that, in this scenario, at the best WE: $\alpha_{\text{MSF}}^{WE} = 1$ (only service MSF is used). Here, the operator revenue is decreasing w.r.t. β_{mc} because of *Hypothesys 4.2*. If $\beta_{mc} \leq 0.18$, at the worst WE: $\alpha_{\text{MS}}^{WE} = 1$ (only service MS is used) and increasing β_{mc} will decrease the operator revenue. If $\beta_{mc} \geq 0.53$, at the worst WE: $\alpha_{\text{M}}^{WE} = 1$ (only service M is used) and and increasing β_{mc} will increase the operator revenue. If $0.18 < \beta_{mc} < 0.53$, the WE is unique and the two curves are superimposed. We observe that the operator can manipulate the equilibrium system loads by tuning the bandwidth allocation policy.

In Fig. 6, we plot the revenue at the optimum prices and both, the worst and best WE (taken for each combination of prices), as functions of β_{mc}. The ranges of prices over which the optimal values were computed are: $p_{\text{M}} \in \{0, 0.1, 0.2, \cdots, 1\}$, $p_{\text{MS}} \in \{0, 0.2, 0.4, \cdots, 3\}$ and $p_{\text{MSF}} \in \{0, 0.2, 0.4, \cdots, 9\}$. The optimal revenue in both cases is decreasing when the fraction of MC bandwidth is increased.

V. Conclusions

We have studied a multi-tier network where an operator deploys heterogeneous technologies (e.g., composed of macro-cells, small-cells and femto-cells). Based on these technologies, the operator can provide to its customers several types of services. We have formalized the joint pricing and bandwidth allocation problem as a Stackelberg game. We have analysed the solution of the corresponding game, both, mathematically and via numerical simulations. We have observed that, in order to provide enhanced services at higher prices, some minimal network resources (e.g., bandwidth) must be allocated to the small-cells and femto-cells. Furthermore, by tuning the pricing and the bandwidth allocation policy, the operator can manipulate the system loads at the equilibrium operating point.

[1] The range of values of β_{mc} is such that *Hypothesis 4.2* is met (see Remark 4.1).

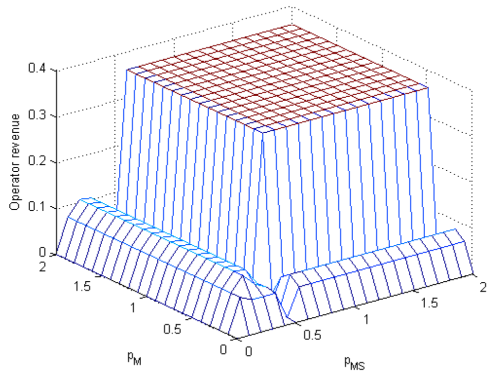

Fig. 1. Operator revenue at the worst WE vs. prices p_{M} and p_{MS} for the scenario: $p_{\mathrm{MSF}} = 0.7$, $(\beta_{mc}, \beta_{sc}, \beta_{fc}) = (0.43, 0.40, 0.17)$.

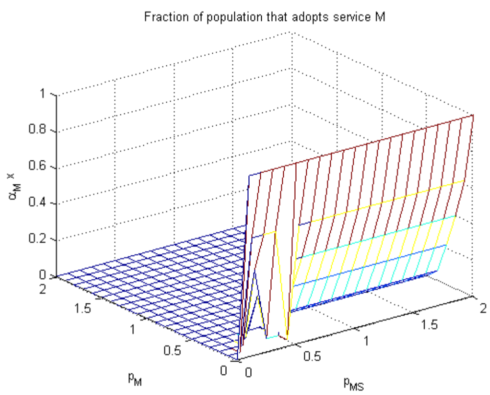

Fig. 2. Fraction of users that adopt service s_{M}: $\alpha_{\mathrm{M}} x$, for $p_{\mathrm{MSF}} = 0.7$ $\gamma_{max} = 1$, $(\beta_{mc}, \beta_{sc}, \beta_{fc}) = (0.43, 0.40, 0.17)$.

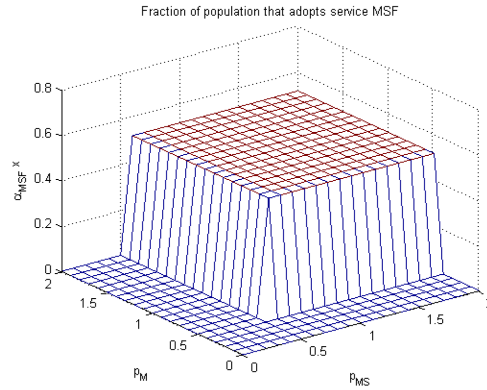

Fig. 4. Fraction of users that adopt service s_{MSF}: $\alpha_{\mathrm{MSF}} x$, for $p_{\mathrm{MSF}} = 0.7$, $(\beta_{mc}, \beta_{sc}, \beta_{fc}) = (0.43, 0.40, 0.17)$.

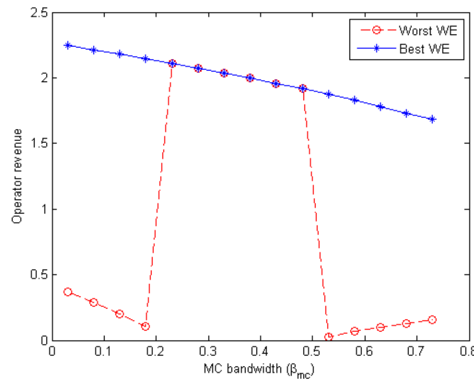

Fig. 5. Operator revenue at the worst and best WE vs. MC bandwidth β_{mc} for $\beta_{fc} = 0.17$, $p_{\mathrm{M}} = 0.5$, $p_{\mathrm{MS}} = 2.0$ and $p_{\mathrm{MSF}} = 4.2$.

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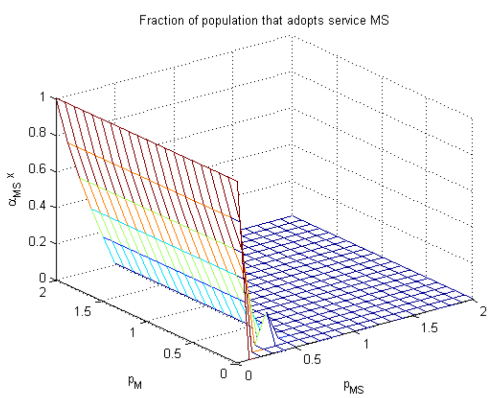

Fig. 3. Fraction of users that adopt service s_{MS}: $\alpha_{\mathrm{MS}} x$, for $p_{\mathrm{MSF}} = 0.7$, $(\beta_{mc}, \beta_{sc}, \beta_{fc}) = (0.43, 0.40, 0.17)$.

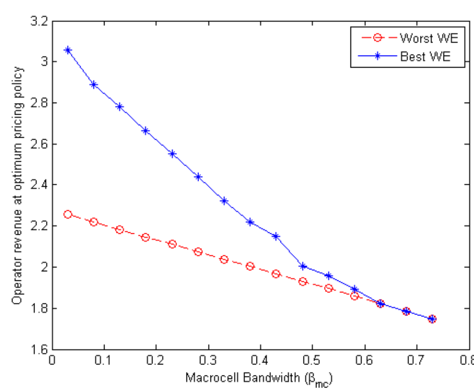

Fig. 6. Optimal operator revenue w.r.t. pricing at the worst and best WE vs. MC bandwidth β_{mc} for $\beta_{fc} = 0.17$.